Aliphatic $\alpha, \beta$-unsaturated carbonyls and benzoquinones ( $\mathrm{C}=\mathrm{C}-\mathrm{C}=\mathrm{O}$ ), and the $\mathrm{C}_{6} \mathrm{H}_{5}-\mathrm{C}=\mathrm{O}(\mathrm{R})$ bond $\left(\mathrm{R}=\mathrm{H}, \mathrm{CH}_{3}\right) \quad 0.89 \pm 0.02(0.09 \pm 0.01) \quad 1.15 \pm 0.03(0.38)$
1,2-Benzoquinone

0.85 (0.06)
$\mathrm{C}_{3}-\mathrm{C}_{4}$ bond
0.93 (0.11)
2. Aromatic molecules (same numbering as above); all $\pi-\pi$ overlap populations for phenyl rings are $0.240 \pm 0.005$ unless noted otherwise

1-Alkylbenzene
1,2-Dimethylbenzene
1,3-Dimethylbenzene
1,4-Dimethylbenzene
1-Formyl- or acetylbenzene
Naphthalene
Azulene
Tropone
C. Carbon bonded to nitrogen and oxygen

Pyrrole
Azabenzenes
Tropone
Benzoquinones
Other carbonyls
D. Nitrogen bonded to nitrogen Pyridazine
$\mathrm{Cl}-\mathrm{C} 2,1.07 \pm 0.01 ; \mathrm{C} 2-\mathrm{C} 3,1.04 \pm 0.01$; C3-C4, 1.06
C1-C2, 1.08; C2-C3, 1.07; C3-C4, 1.04; C4-C5, 1.06
C1-C2, 1.07; C3-C4, 1.08 ; C4-C5, 1.04
C1-C2, 1.07; C2-C3, 1.03
C1-C2, $1.05(0.22)$; C2-C3, 1.04; C3-C4, 1.05
$\mathrm{Cl}-\mathrm{C} 2,1.00(0.18)$; C2-C3, 1.07 (0.29); C1-C6, 1.04 (0.18); C3-C4, 1.01 (0.19)
$\mathrm{C} 1-\mathrm{C} 7,0.94$ (0.09); $\mathrm{C} 1-\mathrm{C} 10,1.00$ (0.21); C1-C2, 1.06 (0.22); C2-C3, 1.02 (0.24); C3-C4, 1.04 (0.23); C9-C10, 1.00 (0.23)
$\mathrm{C} 1-\mathrm{C} 7,0.95$ (0.12); C1-C2, 1.08 (0.29); C2-C3, 0.98 (0.17); C3-C4, 1.09 (0.28)
0.98 (0.17)
$0.96 \pm 0.01(0.22 \pm 0.01)$
0.75 (0.25)
0.79 (0.30)
$0.83 \pm 0.03(0.33 \pm 0.02)$
0.89 (0.18)
${ }^{a}$ Propylene is an exception because of its shorter bond length (see Table I).

# Molecular SCF Calculations on $\mathrm{CH}_{4}, \mathrm{C}_{2} \mathrm{H}_{2}, \mathrm{C}_{2} \mathrm{H}_{4}, \mathrm{C}_{2} \mathrm{H}_{6}, \mathrm{BH}_{3}$, $\mathrm{B}_{2} \mathrm{H}_{6}, \mathrm{NH}_{3}$, and HCN 

William E. Palke and William N. Lipscomb<br>Contribution from the Department of Chemistry, Harvard University, Cambridge, Massachusetts 02138. Received December 27, 1965


#### Abstract

Molecular self-consistent-field calculations with the use of accurate values of all molecular integrals for a minimum basis set have been made for the molecules $\mathrm{CH}_{4}, \mathrm{C}_{2} \mathrm{H}_{2}, \mathrm{C}_{2} \mathrm{H}_{4}, \mathrm{C}_{2} \mathrm{H}_{6}, \mathrm{BH}_{3}, \mathrm{~B}_{2} \mathrm{H}_{6}, \mathrm{NH}_{3}$, and HCN . The SCF wave function, the Hamiltonian matrix, and a population analysis are given for each molecule. Slater exponents are used except for the choice of 1.2 for the exponent of hydrogen.


TThe self-consistent-field (SCF) method based upon linear combinations of atomic orbitals (LCAO) for molecules ${ }^{1}$ is likely to give a good account of those ground-state properties which are not strongly dependent upon electron correlation. Pending development of general molecular integral programs, the level of complexity of ethane has been explored ${ }^{2}$ with the use of a minimum basis set of Slater atomic orbitals on each center. We report here SCF calculations on the ground states of a number of closely related molecules in order to provide as accurate wave functions as can be found with the use of a minimum basis set and accurate (we hope, and indeed expect) values of all molecular integrals. A consistent set of orbital exponents (Slater values for B, C, and N and 1.2 for H ) also allows a comparison of systematic properties (e.g., energy, charge transfer, and Mulliken overlap population) of these ground states. In addition, the Hamiltonian matrices from these SCF results have been employed to obtain atomic parameters for use in the
(1) C. C. J. Roothaan, Rev. Mod. Phys., 23, 69 (1951).
(2) R. M. Pitzer and W. N. Lipscomb, J. Chem. Phys., 39, 1995 (1963).
nonempirical molecular orbital theory described in a preliminary communication ${ }^{3}$ and in the three preceding papers. ${ }^{4-6}$

The exponents for Slater orbitals for B, C, and N were taken from Slater's rules, ${ }^{7}$ rather than from the "best atom" values, because these rules give values which are closer to optimized exponents in $\mathrm{B}_{2} \mathrm{H}_{6},{ }^{3}$ in $\mathrm{BH}_{3},{ }^{8}$ in $\mathrm{CH}_{4},{ }^{9}$ and in many diatomic molecules ${ }^{10}$ of the first row of the periodic table. In the same studies ${ }^{3,8-10}$ the orbital exponents of 1.2 on H produced significantly lower total energies than did the value of 1.0 for H . Explicitly, these exponents are then 1.2 for Is of $\mathrm{H}, 4.7$
(3) M. D. Newton, F. P. Boer, W. E. Palke, and W. N. Lipscomb, Proc. Natl. Acad. Sci. U. S., 53, 1089 (1965).
(4) M. D. Newton, F. P. Boer, and W. N. Lipscomb, J. Am. Chem. Soc., 88, 2353 (1966).
(5) F. P. Boer, M. D. Newton, and W. N. Lipscomb, ibid., 88, 2361 (1966).
(6) M. D. Newton, F. P. Boer, and W. N. Lipscomb, ibid., 88, 2367 (1966).
(7) C. A. Coulson, "Valence," 2nd ed, Oxford University Press, London, 1961, p 40.
(8) W. E. Palke and W. N. Lipscomb, to be published.
(9) R. M. Pitzer, to be published (personal communication).
(10) B. J. Ransil, Rev. Mod. Phys., 32, 245 (1960).

Table I. Wave Function of Methane

| MO | Orbital energy | H1 | H2 | H3 | H4 | Coefficien 1sC | 2 sC | $2 \mathrm{p}_{2} \mathrm{C}$ | $2 \mathrm{p}_{x} \mathrm{C}$ | $2 \mathrm{p}_{y} \mathrm{C}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $1 \mathrm{a}_{1}$ | -11.2714 | 0.0057 | $\bigcirc 0057$ | 0.0057 | 0.0057 | -0.9953 | -0.0275 | 0 | 0 | 0 |
| $2 a_{1}$ | -0.9320 | 0.1590 | 0.1590 | 0.1590 | 0.1590 | -0.2012 | 0.6644 | 0 | 0 | 0 |
| $1 \mathrm{t}_{22}$ | -0.5418 | 0.2850 | -0.2850 | -0.2850 | 0.2850 | 0 | 0 | 0.5943 | 0 | 0 |
| $1 \mathrm{t}_{2 x}$ | -0.5418 | 0.2850 | 0.2850 | -0.2850 | $-0.2850$ | 0 | 0 | 0 | 0. 5943 | 0 |
| $1 \mathrm{t}_{2}{ }^{2}$ | -0.5418 | 0.2850 | -0.2850 | 0.2850 | -0.2850 | 0 | 0 | 0 | 0 | 0.5943 |
| $2 \mathrm{t}_{2 z}$ | 0.6441 | 0.6472 | -0.6472 | -0.6472 | 0.6472 | 0 | 0 | -1.1316 | 0 | 0 |
| $2 \mathrm{t}_{2 x}$ | 0.6441 | 0.6472 | 0.6472 | -0.6472 | -0.6472 | 0 | 0 | 0 | $-1.1316$ | 0 |
| $2 \mathrm{t}_{2} y$ | 0.6441 | 0.6472 | -0.6472 | 0.6472 | -0.6472 | 0 | ${ }^{0}$ | 0 | 0 | $-1.1316$ |
| $3 a_{1}$ | 0.6887 | 0.7108 | 0.7108 | 0.7108 | 0.7108 | 0.2049 | -1.7253 | 0 | 0 | 0 |
|  | Electronic energy Nuclear repulsion |  | $\begin{array}{r} -53.5059 \\ 13.3918 \end{array}$ | Kinetic energy 39.9045 |  |  |  |  |  |  |
|  | Total energy |  | -40.1141 |  |  |  |  |  |  |  |

Table II. Population Analysis for Methane

| Orbital | Population | Orbital | Population |
| :---: | :---: | :---: | :---: |
| H | 0.867 | ${\mathrm{C} 2 \mathrm{p}_{z}}^{1.088}$ |  |
| C 1 s | 1.996 | $\mathrm{C}_{2}$ | 1.088 |
| C 2 s | 1.274 | $\mathrm{C} 2 \mathrm{p}_{y}$ | 1.088 |

the molecule. Hence, no hand manipulation of integrals is required as the complete list is written onto magnetic tape.

All integrals have been computed to an accuracy of five or more decimal places. In checking this over-all program, we have reproduced McLean's results ${ }^{15}$ on

Table III. Hamiltonian Matrix of $\mathrm{CH}_{4}{ }^{a}$

| H 1 | H 2 | H 3 | H 4 | C 1 s | C 2 s | $\mathrm{C}_{2} \mathrm{p}_{z}$ | $\mathrm{C} 2 \mathrm{p}_{x}$ | $\mathrm{C} 2 \mathrm{p}_{v}$ |
| :---: | ---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| -0.507 | -0.274 | -0.274 | -0.274 | -0.787 | -0.730 | -0.253 | -0.253 | -0.253 |
| -0.274 | -0.507 | -0.274 | -0.274 | -0.787 | -0.730 | 0.253 | -0.253 | 0.253 |
| -0.274 | -0.274 | -0.507 | -0.274 | -0.787 | -0.730 | 0.253 | 0.253 | -0.253 |
| -0.274 | -0.274 | -0.274 | -0.507 | -0.787 | -0.730 | -0.253 | 0.253 | 0.253 |
| -0.787 | -0.787 | -0.787 | -0.787 | -11.268 | -2.640 | 0 | 0 | 0 |
| -0.730 | -0.730 | -0.730 | -0.730 | -2.640 | -1.432 | 0 | 0 | 0 |
| -0.253 | 0.253 | 0.253 | -0.253 | 0 | 0 | -0.349 | 0 | 0 |
| -0.253 | -0.253 | 0.253 | 0.253 | 0 | 0 | 0 | -0.349 | 0 |
| -0.253 | 0.253 | -0.253 | 0.253 | 0 | 0 | 0 | 0 | -0.349 |

${ }^{a}$ All the Hamiltonian matrices are given as square matrices with the rows in the same order as the columns. The labels on the rows are omitted.
for 1 s of $\mathrm{B}, 1.3$ for 2 s and 2 p of $\mathrm{B}, 5.7$ for 1 s of $\mathrm{C}, 1.625$ for 2 s and 2 p of $\mathrm{C}, 6.7$ for 1 s of N , and 1.95 for 2 s and 2 p of N. Atomic units of distance, $a=0.529172 \mathrm{~A}$, and of energy, $E=2 R=27.2098 \mathrm{ev}$, have been used throughout this study.

## Procedure and Results

Computer Programs. Most of the required molecular integrals were computed with the use of programs based upon $\zeta$-function methods. ${ }^{11}$ Some of these programs were written at the Solid State and Molecular Theory Group at MIT, while others were developed here in conjunction with the MIT group as described earlier. ${ }^{2,12}$ Four-center integrals over two $p$ orbitals were computed from Shavitt's program ${ }^{13}$ which employs the Gaussian transform procedure developed by Shavitt, Kern, and Karplus. ${ }^{14}$ Control programs have now been written to produce these integrals in an optimum order, and to transform from the coordinates in which they are computed to the coordinate system of

[^0]acetylene, Merrifield's results ${ }^{16}$ on water, and the ethane calculation ${ }^{2}$ all to four decimal places or better. Also we have reproduced Foster and Boys' results ${ }^{17}$ for the energy to three decimal places and the molecular orbital coefficients to two decimal places on formaldehyde, for which integrals were believed to be good only to $\pm 0.002$. Several of Ransil's diatomic calculations ${ }^{18}$ have been reproduced, and the earlier calculations for hydrogen cyanide ${ }^{19}$ and for ethylene ${ }^{20}$ (from a Gaussian basis) seem quite reasonable in comparison with our results. Thus, we feel that the programs are working correctly.

Methane. Atomic coordinates for $\mathrm{H}_{1}$ at aaa, $\mathrm{H}_{2}$ at $a \bar{a} \bar{a}, \mathrm{H}_{3}$ at $\bar{a} a \bar{a}$, and $\mathrm{H}_{4}$ at $\bar{a} \bar{a} a$, where $a=1.1930942 \mathrm{au}$, were obtained by inscribing the tetrahedral molecule inside a cube of side $2 a$ and scaling ${ }^{21}$ with the use of the
(15) A. D. McLean, ibid., 32, 1595 (1960).
(16) D. Merrifield, personal communication of unpublished results.
(17) (a) J. M. Foster and S. F. Boys, Rev. Mod. Phys., 32, 303 (1960). (b) P. L. Goodfriend, F. W. Birss, and A. B. F. Duncan, ibid., 32, 307 (1960), have also produced results for formaldehyde, but our recalculation of this function has yielded somewhat different results (M. Newton and W. E. Palke, to be published).
(18) B. J. Ransil, Reo. Mod. Phys., 32, 245 (1960).
(19) A. D. McLean, J. Chem. Phys., 37, 627 (1962).
(20) J. W. Moscowitz, ibid., 43, 60 (1965).
(21) In order to permit detailed future checks on our results, a number of molecular parameters, particularly input geometrical coordinates, are given to a number of decimal places far beyond experimental significance.
Table IV. Wave Function of Acetylene

equilibrium $\mathrm{C}-\mathrm{H}$ bond distance. ${ }^{22}$ The wave function and energies are shown in Table I, the population analysis in Table II, and the SCF Hamiltonian matrix in Table III. We were unable to reproduce Sinai's results, ${ }^{23}$ but we did obtain with the use of his parameters ( $\mathrm{C}-\mathrm{H}=2 \mathrm{au}$ and exponent 1.0 for H ) a molecular energy of -40.0444 au , which has inclependently been obtained by Pitzer. ${ }^{24}$ Our results yield a charge of +0.113 on each H , and a C-H overlap population of 0.780 . The wave function and energies compare well with results obtained by Pitzer, ${ }^{24}$ who found optimum exponents of 2 s and 2 p of C to be 1.76 . Comparison of our results with those of Woznick ${ }^{25}$ is reasonable, but more complex; he used a combination of three ls functions with different exponents for 1 s of C , but our H exponent is more nearly optimal than is his choice of 1.0 or 1.5 in his "minimal representation." Also his choices of 1.5525 for 2 s of C and 1.5793 for 2 p of C are not as close to the optimal value as our Slater exponent of 1.625 . Hence, for the most nearly comparable calculations, our energy is lower than those ( -40.001 to -40.073 au ) for his minimal representation, but Woznick's calculations with a larger basis set gave energies as low as -40.181 au , which is a considerable further improvement over our results.

Acetylene. Coordinates of the ground-state equilibrium distances ${ }^{26}$ are $z=0$ for $\mathrm{C} 1,2.281$ for C 2 , -2.002 for H 1 , and 4.283 for H2. Results are given in Table IV for wave function and energy, in Table V

Table V. Population Analysis for Acetylene

| Orbital | Population | Orbital | Population |
| :---: | :---: | :---: | :---: |
| H | 0.812 | ${\mathrm{C} 2 \mathrm{p}_{z}}^{1.086}$ |  |
| C 1 s | 1.997 | ${\mathrm{C} 2 \mathrm{p}_{x}}^{\mathrm{C}}$ | 1.0 |
| C 2 s | 1.105 | $\mathrm{C} \mathrm{p}_{y}$ | 1.0 |

for the population analysis, and in Table VI for the SCF Hamiltonian matrix. The resulting charge on H is +0.188 , and overlap populations are 0.820 for $\mathrm{C}-\mathrm{H}$, 0.874 for $\mathrm{C}-\mathrm{C}_{\sigma}$, and 1.001 for $\mathrm{C}-\mathrm{C}_{\pi}$. Values from McLean's "best atom" calculation ${ }^{27}$ are +0.213 for the charge of $H$, and $0.824,0.892$, and 1.051 for the $\mathrm{C}-\mathrm{H}, \mathrm{C}-\mathrm{C}_{\sigma}$, and $\mathrm{C}-\mathrm{C}_{\pi}$ overlap populations, respectively. A calculation by Moscowitz, ${ }^{20}$ who used an extended set of Gaussian functions, yields a charge of +0.205 on H , and total energy of -76.741 au , which is lower than our value ( -76.6165 au ) from a minimum basis set of Slater orbitals.

Ethylene. Coordinates for the equilibrium geometry ${ }^{22}$ of the ground state are given in Table VII, wave function and energies in Table VIII, population analysis in Table X, and the SCF Hamiltonian matrix in Table IX. The charge on H is +0.140 , which is to be compared to the charge of +0.180 obtained by Moscowitz. ${ }^{20}$ We obtain overlap populations of 0.812 for $\mathrm{C}-\mathrm{H}, 0.791$ for $\mathrm{C}-\mathrm{C}_{\sigma}$, and 0.420 for $\mathrm{C}-\mathrm{C}_{\pi}$.

[^1]Table VI. Hamiltonian Matrix of $\mathrm{C}_{2} \mathrm{H}_{2}$

| H 1 | H 2 | 1 C 1 s | 1 C 2 s | $1 \mathrm{C} 2 \mathrm{p}_{z}$ | $1 \mathrm{C} 2 \mathrm{p}_{x}$ | $1 \mathrm{C} 2 \mathrm{p}_{y}$ | 2 C 1 s | 2 C 2 s | $2 \mathrm{C} 2 \mathrm{p}_{2}$ | $2 \mathrm{C} 2 \mathrm{p}_{x}$ | $2 \mathrm{C} 2 \mathrm{p}_{y}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| -0.564 | -0.032 | -0.848 | -0.771 | 0.436 | 0 | 0 | -0.059 | -0.203 | 0.251 | 0 | 0 |
| -0.032 | -0.564 | -0.059 | -0.203 | -0.251 | 0 | 0 | -0.848 | -0.771 | -0.436 | 0 | 0 |
| -0.848 | -0.059 | -11.294 | -2.645 | -0.005 | 0 | 0 | -0.003 | -0.775 | 1.298 | 0 | 0 |
| -0.771 | -0.203 | -2.645 | -1.460 | -0.184 | 0 | 0 | -0.775 | -0.940 | 0.783 | 0 | 0 |
| 0.436 | -0.251 | -0.005 | -0.184 | -0.749 | 0 | 0 | -1.298 | -0.783 | 0.271 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | -0.169 | 0 | 0 | 0 | 0 | -0.372 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | -0.169 | 0 | 0 | 0 | 0 | -0.372 |
| -0.059 | -0.848 | -0.003 | -0.775 | -1.298 | 0 | 0 | -11.294 | -2.645 | 0.005 | 0 | 0 |
| -0.203 | -0.771 | -0.775 | -0.940 | -0.783 | 0 | 0 | -2.645 | -1.460 | 0.184 | 0 | 0 |
| 0.251 | -0.436 | 1.298 | 0.783 | 0.271 | 0 | 0 | 0.005 | 0.184 | -0.749 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | -0.372 | 0 | 0 | 0 | 0 | -0.169 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | -0.372 | 0 | 0 | 0 | 0 | -0.169 |

Table VII. Geometry of Ethylene ${ }^{a}$

| Atom | $x$ | $y$ | $z$ |
| :---: | :---: | :---: | :---: |
| C 1 | 0 | 0 | 0 |
| C 2 | 0 | 0 | 2.5511602 |
| H 1 | 1.7511301 | 0 | -1.0110153 |
| H 2 | -1.7511301 | 0 | -1.0110153 |
| H 3 | 1.7511301 | 0 | 3.5621755 |
| H 4 | -1.7511301 | 0 | 3.5621755 |

${ }^{a}$ This coordinate system differs from the recommendation that the molecule lies in the $y z$ plane: R. S. Mulliken, J. Chem. Phys., 23, 1997 (1955).

Ethane. The atomic coordinates ${ }^{28}$ listed in Table XI are identical with those of the previous study ${ }^{2}$ which used 1.0 as the orbital exponent for $H$. The present study is concerned only with the staggered geometry. Wave functions and energies (Table XX), population analysis (Table XII), and the SCF Hamiltonian matrix (Table XXI) are all reasonable in comparison with the earlier results. ${ }^{2}$ In the present study a charge of +0.124 on H , a $\mathrm{C}-\mathrm{H}$ overlap population of 0.785 , and a $\mathrm{C}-\mathrm{C}$ overlap population of 0.710 may be compared with the earlier results (with a different exponent for $H$ ) of $+0.114,0.777$, and 0.704 for these three quantities, respectively.

Borane ( $\mathrm{BH}_{3}$ ). Because $\mathrm{BH}_{3}$ dimerizes rapidly to $\mathrm{B}_{2} \mathrm{H}_{6}$, no direct evidence for the geometry of $\mathrm{BH}_{3}$ is known, but a planar structure is most reasonable ${ }^{29}$ and is supported by a theoretical study. ${ }^{30}$ In a series of calculations to be reported elsewhere, ${ }^{8}$ detailed variation of orbital exponents and $\mathrm{B}-\mathrm{H}$ distance in $\mathrm{BH}_{3}$ has yielded an optimum $\mathrm{B}-\mathrm{H}$ distance of 2.25 au , which is very close to the $\mathrm{B}-\mathrm{H}$ (terminal) distance of 2.26 au in diborane. ${ }^{31}$ Coordinates in the $x, y$ plane are Hl at $x=2.25$ and $y=0, \mathrm{H} 2$ at $x=-1.125$ and $y=$ 1.9485572, and H3 at $x=-1.125$ and $y=-1.9485572$. Wave functions and energies (Table XIII), population analysis (Table XV), and SCF Hamiltonian matrix (Table XIV) yield a charge of +0.057 on each $H$, and a B-H overlap population of 0.837 .

Diborane. The molecular geometry (Table XVII) is taken from the recent electron diffraction study of Bartell and Carroll. ${ }^{31}$ Wave functions, energies, population analysis, and SCF Hamiltonian matrix are given in Tables XXII, XVIII, and XXIV. Our results are substantially different from those of Yamazaki, ${ }^{32}$ and

[^2]from the earlier four-center results of Hamilton, ${ }^{33}$ both of whom found that the charge on H(bridge) was about -0.2 . In contrast, we find the charge on the bridge H atoms is +0.099 and on the terminal H atoms is +0.092 . When the orbital exponents on $H$ and $B$ have been optimized, as reported previously, ${ }^{3}$ the charges are -0.001 on H (bridge) and -0.032 on H (terminal). Overlap populations of 0.864 for $\mathrm{B}-\mathrm{H}-$ (terminal), $\mathrm{B}-\mathrm{H}$ (bridge) of 0.399 , and $\mathrm{B}-\mathrm{B}$ of 0.291 are especially interesting in view of the moderately strong direct B-B interaction. ${ }^{34}$

Ammonia. Coordinates are chosen with N at the origin and three H atoms in the $z=0.71996$ plane, with H1 at $x=1.776$ and $y=0, \mathrm{H} 2$ at $x=-0.888$ and $y=1.538$, and H3 at $x=-0.888$ and $y=-1.538$. Wave functions and energies (Table XXV), population analysis (Table XVI), and the SCF Hamiltonian matrix (Table XXVI) lead to a charge of +0.155 on each H , and a $\mathrm{N}-\mathrm{H}$ overlap population of 0.677 . The molecular dipole moment of 1.72 D . is computed from the detailed wave function, not from the charges of the population analysis. The observed value ${ }^{35}$ is 1.47 D . Orbital energies are in the same order and in qualitative agreement with results obtained by Kaplan, ${ }^{36}$ who obtained an energy of -56.266 au and a dipole moment of 1.82 D. with the use of Hartree-Fock atomic orbitals and a single determinantal wave function. On the other hand, we have repeated the calculation with the exponent of 1.0 for H in an attempt to reproduce Duncan's results, ${ }^{37}$ but we obtained an energy of -55.9645 au. Although our values of one- and two-center integrals agree with his values, our threeand four-center integrals are not in agreement with his.

Hydrogen Cyanide. The equilibrium geometry ${ }^{36}$ has C at the origin, H at $z=-2.0$, and N at 2.187. The wave function and energies (Table XXIII), population analysis (Table XIX), and SCF Hamiltonian matrix (Table XXVII) yield a charge of +0.216 on $\mathrm{H},-0.141$ on C , and -0.075 on N . The dipole moment computed from the detailed wave function is 2.11 D ., as compared with the observed value ${ }^{38}$ of 2.95 D .

## (32) M. Yamazaki, ibid., 27, 1401 (1957).

(33) W. C. Hamilton, Proc. Roy. Soc. (London), A235, 395 (1956).
(34) A molecular orbital calculation for diborane using an extended set of gaussian orbitals has recently been completed by L. Burnelle and J. J. Kaufman (J. Chem. Phys, 43, 3540 (1965)). They obtained a charge of +0.045 on the terminal H and a nearly neutral bridge H . Their B-H overlap populations agree well with ours; however, they obtain a B-B overlap population of 0.064 which is surprisingly low.
(35) A. L. McClellan, "Tables of Experimental Dipole Moments,"
w. H. Freeman and Co., San Francisco, Calif., 1963, p 27.
(36) H. Kaplan, J. Chem. Phys., 26, 1704 (1957).
(37) A. B. F. Duncan, ibid., 27, 423 (1957).
(38) Reference $35, \mathrm{p} 39$.

Table VIII. Wave Function of Ethylene


Table IX. Hamiltonian Matrix of $\mathrm{C}_{2} \mathrm{H}_{4}$

| H1 | H2 | H3 | H4 | 1C1s | 1C2s | $1 \mathrm{C} 2 \mathrm{p}_{2}$ | $1 \mathrm{C} 2 \mathrm{p}_{x}$ | $1 \mathrm{C} 2 \mathrm{p}_{y}$ | 2C1s | 2C2s | $2 \mathrm{C} 2 \mathrm{p}_{2}$ | $2 \mathrm{C} 2 \mathrm{p}_{x}$ | $2 \mathrm{C} 2 \mathrm{p}_{y}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| -0.537 | -0.260 | -0.088 | -0.049 | -0.829 | -0.758 | 0.206 | -0.387 | 0 | -0.084 | -0.232 | 0.250 | -0.080 | 0 |
| $-0.260$ | -0.537 | -0.049 | -0.088 | -0.829 | -0.758 | 0.206 | 0.387 | 0 | -0.084 | -0.232 | 0.250 | 0.080 | 0 |
| $-0.088$ | $-0.049$ | $-0.537$ | $-0.260$ | -0.084 | -0.232 | $-0.250$ | $-0.080$ | 0 | -0.829 | -0.758 | $-0.206$ | $-0.387$ | 0 |
| -0.049 | $-0.088$ | $-0.260$ | $-0.537$ | -0.084 | -0.232 | $-0.250$ | 0.080 | 0 | -0.829 | $-0.758$ | -0.206 | 0.387 | 0 |
| $-0.829$ | -0.829 | -0.084 | -0.084 | -11.284 | -2.643 | $-0.002$ | 0 | 0 | -0.001 | $-0.567$ | 0.956 | 0 | 0 |
| -0.758 | -0.758 | $-0.232$ | -0.232 | -2.643 | $-1.463$ | -0.106 | 0 | 0 | -0.567 | -0.772 | 0.680 | 0 | 0 |
| 0.206 | 0.206 | $-0.250$ | -0.250 | -0.002 | -0.106 | -0.549 | 0 | 0 | -0.956 | $-0.680$ | 0.294 | 0 | 0 |
| -0.387 | 0.387 | $-0.080$ | 0.080 | 0 | 0 | 0 | $-0.411$ | 0 | 0 | 0 | 0 | $-0.232$ | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | $-0.146$ | 0 | 0 | 0 | 0 | $-0.324$ |
| $-0.084$ | $-0.084$ | $-0.829$ | -0.829 | $-0.001$ | $-0.567$ | -0.956 | 0 | 0 | -11.284 | -2.643 | 0.002 | 0 | 0 |
| -0.232 | -0.232 | -0.758 | -0.758 | $-0.567$ | -0.772 | -0.680 | 0 | 0 | -2.643 | $-1.463$ | 0.106 | 0 | 0 |
| 0.250 | 0.250 | -0.206 | -0.206 | 0.956 | 0.680 | 0.294 | 0 | 0 | 0.002 | 0.106 | -0.549 | 0 | 0 |
| -0.080 | 0.080 | -0.387 | 0.387 | 0 | 0 | 0 | $-0.232$ | 0 | 0 | 0 | 0 | $-0.411$ | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -0.324 | 0 | 0 | 0 | 0 | -0.146 |

Table X. Population Analysis for Ethylene

| Orbital | Population | Orbital | Population |
| :---: | :---: | :---: | :---: |
| H | 0.860 | ${\mathrm{C} 2 \mathrm{p}_{z}}^{\mathrm{C}} \mathrm{s}$ | 1.996 |
| $\mathrm{C} 2 \mathrm{p}_{x}$ | 1.013 |  |  |
| C 2 s | 1.197 | $\mathrm{C} 2 \mathrm{p}_{y}$ | 1.072 |

Table XI. Geometry of Ethane

| Atom | $x$ | $y$ | $z$ |
| :---: | :---: | :---: | :---: |
| C1 | 0 | 0 | 0 |
| C2 | 0 | 0 | 2.9158761 |
| H1 | 1.9616301 | 0 | -0.6991481 |
| H2 | -0.9808151 | 1.6988215 | -0.6991481 |
| H3 | -0.9808151 | -1.6988215 | -0.6991481 |
| H4 | -1.9616301 | 0 | 3.6150242 |
| H5 | 0.9808151 | -1.6988215 | 3.6150242 |
| H6 | 0.9808151 | 1.6988215 | 3.6150242 |

Table XII. Population Analysis for Ethane

| Orbital | Population |
| :--- | :---: |
| H | 0.876 |
| C 1 s | 1.995 |
| C 2 s | 1.248 |
| $\mathrm{C} 2 \mathrm{p}_{2}$ | 0.981 |
| $\mathrm{C} 2 \mathrm{p}_{x}$ | 1.074 |
| $\mathrm{C} 2 \mathrm{p}_{y}$ | 1.074 |

Table XVII. Geometry of Diborane

| Atom | $x$ | $y$ | $z$ |
| :---: | :---: | :---: | ---: |
| B1 | 0 | 0 | 0 |
| B2 | 0 | 0 | 3.3542969 |
| H1 | 1.9473999 | 0 | -1.1471018 |
| H2 | -1.9473999 | 0 | -1.1471018 |
| H3 | 1.9473999 | 0 | 4.5013987 |
| H4 | -1.9473999 | 0 | 4.5013987 |
| H5 | 0 | 1.8947127 | 1.6771484 |
| H6 | 0 | -1.8947127 | 1.6771484 |

Table XVIII. Population Analysis for Diborane

| Orbital | Population | Orbital | Population |
| :---: | :---: | :---: | :---: |
| H1 | 0.908 | $\mathrm{~B} 2 \mathrm{p}_{z}$ | 0.727 |
| H5 | 0.901 | ${\mathrm{~B} 2 \mathrm{p}_{x}}^{\text {B1s }}$ | 1.995 |
| B2s | 1.025 |  | 1.085 |
| B2 | 0.451 |  |  |

Table XIX. Population Analysis for Hydrogen Cyanide

| Orbital | Population | Orbital | Population |
| :--- | :---: | :--- | :---: |
| H | 0.784 | N 1 s | 1.998 |
| C 1 s | 1.997 | N 2 s | 1.764 |
| C 2 s | 1.090 | $\mathrm{~N} 2 \mathrm{p}_{z}$ | 1.351 |
| ${\mathrm{C} 2 \mathrm{p}_{z}}^{\mathrm{C}_{2}}$ | 1.014 | $\mathrm{~N} 2 \mathrm{p}_{x}$ | 0.981 |
| $\mathrm{C} 2 \mathrm{p}_{y}$ | 1.019 | $\mathrm{~N} 2 \mathrm{p}_{y}$ | 0.981 |

Table XIII. Wave Function of Borane

| MO | Orbital energy | H1 | H2 | H3 | $-\mathrm{Coet}$ | 2 sB | $2 \mathrm{p}_{2} \mathrm{~B}$ | $2 \mathrm{p}_{x} \mathrm{~B}$ | $2 \mathrm{p}_{\nu} \mathrm{B}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $1 a_{1}{ }^{\prime}$ | -7.6795 | 0.0048 | 0.0048 | 0.0048 | -0.9960 | -0.0233 | 0 | 0 | 0 |
| $2 a_{1}{ }^{\prime}$ | -0.7129 | 0.2457 | 0.2457 | 0.2457 | -0.1902 | 0.5932 | 0 | 0 | 0 |
| $1 \mathrm{e}_{x}$ | -0.5104 | 0.4614 | -0.2307 | -0.2307 | 0 | 0 | 0 | 0.5545 | 0 |
| $1 \mathrm{e}_{y}$ | -0.5104 | 0 | 0.3996 | -0.3996 | 0 | 0 | 0 | 0 | 0.5545 |
| $1 a_{1}{ }^{\prime \prime}$ | 0.1139 | 0 | 0 | 0 | 0 | 0 | 1.0 | 0 | 0 |
| $3 a_{1}{ }^{\prime}$ | 0.6094 | 0.8484 | 0.8484 | 0.8484 | 0.1701 | -1.6279 | 0 | 0 | 0 |
| $2 \mathrm{e}_{x}$ | 0.6475 | 1.1296 | -0.5648 | $-0.5648$ | 0 | 0 | 0 | $-1.2871$ | 0 |
| $2 \mathrm{e}_{y}$ | 0.6475 | 0 | $0.9783$ | $-0.9783$ | 0 | 0 | 0 | 0 | $-1.2871$ |
|  | Electronic energy <br> Nuclear repulsion |  | $\begin{array}{r} -33.7742 \\ 7.4365 \end{array}$ | Kinetic energy 26.1784 |  |  |  |  |  |
|  | Total energy |  | -26.3377 |  |  |  |  |  |  |

Table XIV. Hamiltonian Matrix of $\mathrm{BH}_{3}$

| H 1 | H 2 | H 3 | B 1 s | B 2 s | $\mathrm{~B} 2 \mathrm{p}_{2}$ | $\mathrm{~B} 2 \mathrm{p}_{x}$ | $\mathrm{~B} 2 \mathrm{p}_{y}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| -0.465 | -0.176 | -0.176 | -0.584 | -0.569 | 0 | 0 |  |
| -0.176 | -0.465 | -0.176 | -0.584 | -0.569 | 0 | -0.406 |  |
| -0.176 | -0.176 | -0.465 | -0.584 | -0.569 | 0 | 0.203 | -0.352 |
| -0.584 | -0.584 | -0.584 | -7.677 | -1.719 | 0 | 0.203 | 0 |
| -0.569 | -0.569 | -0.569 | -1.719 | -0.969 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0.114 | 0 | 0 |
| -0.406 | 0.203 | 0.203 | 0 | 0 | 0 | 0 | 0 |
| 0 | -0.352 | 0.352 | 0 | 0 | 0 | 0 | 0 |

Table XV. Population Analysis for Borane

| Orbital | Population | Orbital | Population |
| :---: | :---: | :---: | :---: |
| H | 0.943 | ${\mathrm{~B} 2 \mathrm{p}_{z}}$ |  |
| B1s | 1.997 | ${\mathrm{~B} 2 \mathrm{p}_{x}}$ | 1.027 |
| B2s | 1.121 | ${\mathrm{~B} 2 \mathrm{p}_{y}}^{1.027}$ |  |

Table XVI. Population Analysis for Ammonia

| Orbital | Population | Orbital | Population |
| :---: | :---: | :---: | :---: |
| H | 0.845 | $\mathrm{~N} 2 \mathrm{p}_{z}$ | 1.760 |
| N 1 s | 1.997 | $\mathrm{~N} 2 \mathrm{p}_{x}$ | 1.056 |
| N 2 s | 1.598 | $\mathrm{~N} 2 \mathrm{p}_{y}$ | 1.056 |

McLean, ${ }^{19}$ in his "best atom" calculation on HCN, obtained charges of 0.24 on $\mathrm{H},-0.16$ on C , and -0.08 on N , in excellent agreement with our values. Our $\pi$-electron transfer of 0.038 electron (negative charge) from N to C compares well with his transfer of 0.06 electron in the same direction. Finally, our overlap populations of 0.800 for $\mathrm{C}-\mathrm{H}, 0.544$ for $\mathrm{C}-\mathrm{N}_{\sigma}$, and 0.918 for $\mathrm{C}-\mathrm{N}_{\pi}$ agree well with his values of 0.801 , 0.520 , and 0.953 , respectively.

## Discussion

The most extended series of comparable molecules in this paper is the group, $\mathrm{C}_{2} \mathrm{H}_{2}, \mathrm{C}_{2} \mathrm{H}_{4}, \mathrm{C}_{2} \mathrm{H}_{6}$, and $\mathrm{CH}_{4}$.

Table XX. Wave Function of Ethane


Table XXI. Hamiltonian Matrix of $\mathrm{C}_{2} \mathrm{H}_{6}$

| H1 | H2 | H3 | H4 | H5 | H6 | 1 Cl s | 1C2s | $1 \mathrm{C} 2 \mathrm{p}_{2}$ | $1 \mathrm{C} 2 \mathrm{p}_{x}$ | $1 \mathrm{C} 2 \mathrm{p}_{y}$ | 2 Cls | 2C2s | $2 \mathrm{~S} 2 \mathrm{p}_{z}$ | $2 \mathrm{C} 2 \mathrm{p}_{x}$ | $2 \mathrm{C} 2 \mathrm{p}_{\nu}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| -0.501 | $-0.268$ | -0.268 | $-0.042$ | $-0.076$ | $-0.076$ | $-0.773$ | $-0.727$ | 0.139 | -0.413 | 0. | -0.071 | -0.190 | 0.198 | $-0.071$ | 0 |
| -0.268 | -0. 501 | -0.268 | -0.076 | -0.043 | $-0.076$ | -0.773 | -0.727 | 0.139 | 0.207 | $-0.358$ | -0.071 | $-0.190$ | 0.198 | 0.035 | -0.061 |
| -0.268 | -0.268 | -0.501 | -0.076 | -0.076 | -0.043 | -0.773 | -0.727 | 0.139 | 0.207 | 0.358 | -0.071 | $-0.190$ | 0.198 | 0.035 | 0.061 |
| -0.043 | -0.076 | -0.076 | -0. 501 | -0.268 | -0.268 | -0.071 | $-0.190$ | -0.198 | 0.071 | 0 | -0.773 | $-0.727$ | -0.139 | 0.413 | 0 |
| -0.076 | -0.043 | -0.076 | -0.268 | -0.501 | -0.268 | -0.071 | $-0.190$ | -0.198 | -0.035 | 0.061 | -0.773 | -0.727 | -0.139 | -0.207 | 0.358 |
| -0.076 | -0.076 | -0.043 | -0.268 | -0.268 | -0.501 | -0.071 | $-0.190$ | -0.198 | -0.035 | -0.061 | $-0.773$ | $-0.727$ | -0.139 | -0.207 | -0.358 |
| -0.773 | -0.773 | -0.773 | -0.071 | $-0.071$ | $-0.071$ | -11.277 | -2.641 | 0 | 0 | 0 | 0 | -0.364 | 0.619 | 0 | 0 |
| -0.727 | -0.727 | -0.727 | -0.190 | -0.190 | -0.190 | -2.641 | -1.449 | -0.029 | 0 | 0 | $-0.364$ | -0.575 | 0.540 | 0 | 0 |
| 0.139 | 0.139 | 0.139 | -0.198 | -0.198 | -0.198 | 0 | -0.029 | -0.386 | 0 | 0 | -0.619 | -0.540 | 0.301 | 0 | 0 |
| -0.413 | 0.207 | 0.207 | 0.071 | -0.035 | -0.035 | 0 | 0 | 0 | $-0.354$ | 0 | 0 | 0 | 0 | -0.168 | 0 |
| 0 | -0.358 | 0.358 | 0 | 0.061 | -0.061 | 0 | 0 | 0 | 0 | $-0.354$ | 0 | 0 | 0 | 0 | -0.168 |
| -0.071 | -0.071 | -0.071 | -0.773 | -0.773 | -0.773 | 0 | -0.364 | -0.619 | 0 | 0 | -11.277 | -2.641 | 0 | 0 | 0 |
| -0.190 | -0.190 | -0.190 | -0.727 | -0.727 | -0.727 | -0.364 | -0.575 | -0.540 | 0 | 0 | -2.641 | -1.449 | 0.029 | 0 | 0 |
| 0.198 | 0.198 | 0.198 | -0.139 | -0.139 | -0.139 | 0.619 | 0.540 | 0.301 | 0 | 0 | 0 | 0.029 | -0.386 | 0 | 0 |
| -0.071 | 0.035 | 0.035 | 0.413 | -0.207 | -0.207 | 0 | 0 | 0 | $-0.168$ | 0 | 0 | 0 | 0 | -0.354 | 0 |
| 0 | -0.061 | 0.061 | 0 | 0.358 | -0.358 | 0 | 0 | 0 | 0 | -0.168 | 0 | 0 | 0 | 0 | -0.354 |

Table XXII. Wave Function of Diborane


Table XXIII. Wave Function of Hydrogen Cyanide

Table XXIV. Hamiltonian Matrix of $\mathrm{B}_{2} \mathrm{H}_{6}$

| H1 | H2 | H3 | H4 | H5 | H6 | 1 Bls | 182s | $1 \mathrm{~B} 2 \mathrm{p}_{z}$ | $1 \mathrm{~B} 2 \mathrm{p}_{x}$ | $1 \mathrm{~B} 2 \mathrm{p}_{\text {, }}$ | 2BIs | 2B2s | 2B2p ${ }_{\text {z }}$ | 2B2p ${ }_{x}$ | 2B2p ${ }_{y}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| -0.464 | -0.179 | -0.032 | 0.018 | -0.175 | -0.175 | $-0.580$ | -0.584 | 0.188 | -0.355 | 0 | -0.026 | -0.134 | 0.166 | -0.052 | 0 |
| -0.179 | -0.464 | -0.018 | -0.032 | -0.175 | -0.175 | -0.580 | -0. 584 | 0.188 | 0.355 | 0 | -0.026 | -0.134 | 0.166 | 0.052 | 0 |
| -0.032 | -0.018 | -0.464 | -0.179 | -0.175 | -0.175 | -0.026 | -0.134 | -0.166 | -0.052 | 0 | -0.580 | -0.584 | -0.188 | -0.355 | 0 |
| -0.018 | -0.032 | -0.179 | -0.464 | -0.175 | -0.175 | -0.026 | -0.134 | -0.166 | 0.052 | 0 | -0.580 | -0.584 | -0.188 | 0.355 | 0 |
| -0.175 | -0.175 | -0. 175 | -0.175 | -0. 593 | -0.217 | -0.426 | -0.564 | -0.372 | 0 | -0.301 | -0.426 | -0.564 | 0.372 | 0 | -0.301 |
| -0.175 | -0.175 | -0.175 | -0.175 | -0.217 | -0.593 | -0.426 | -0.564 | -0.372 | 0 | 0.301 | -0.426 | -0.564 | 0.372 | 0 | 0.301 |
| $-0.580$ | -0. 580 | -0.026 | -0.026 | -0.426 | -0.426 | -7.706 | -1.728 | -0.009 | 0 | 0 | 0 | -0.317 | 0.538 | 0 | 0 |
| -0.584 | -0.584 | -0.134 | -0.134 | -0.564 | -0.564 | -1.728 | -1.081 | -0.104 | 0 | 0 | -0.317 | -0.536 | 0.497 | 0 | 0 |
| 0.188 | 0.188 | -0.166 | -0.166 | -0. 372 | -0.372 | -0.009 | -0.104 | -0.439 | 0 | 0 | -0.538 | -0. 497 | 0.281 | 0 | 0 |
| -0.355 | 0.355 | -0.052 | 0.052 | 0 | 0 | 0 | 0 | 0 | -0.388 | 0 | 0 | 0 | 0 | -0.160 | 0 |
| 0 | 0 | 0 | 0 | -0.301 | 0.301 | 0 | 0 | 0 | 0 | -0.183 | 0 | 0 | 0 | 0 | $-0.280$ |
| -0.026 | -0.026 | -0.580 | -0.580 | -0.426 | -0.426 | 0 | -0.317 | -0.538 | 0 | 0 | -7.706 | -1.728 | 0.009 | 0 | 0 |
| -0.134 | -0.134 | -0.584 | -0.584 | -0.564 | -0.564 | -0.317 | -0.536 | -0.497 | 0 | 0 | -1.728 | -1.081 | 0.104 | 0 | 0 |
| 0.166 | 0.166 | -0.188 | -0.188 | 0.372 | 0.372 | 0.538 | 0.497 | 0.281 | 0 | 0 | 0.009 | 0.104 | -0.439 | 0 | 0 |
| $-0.052$ | 0.052 | $-0.355$ | 0.355 | 0 | 0 | 0 | 0 | 0 | $-0.160$ | 0 | 0 | 0 | 0 | -0.388 | 0 |
| 0 | 0 | 0 | 0 | -0.301 | 0.301 | 0 | 0 | 0 | 0 | -0.280 | 0 | 0 | 0 | 0 | $-0.183$ |

As saturation increases, the negative charge on C varies monotonically as -0.188 in $\mathrm{C}_{2} \mathrm{H}_{2},-0.280$ in $\mathrm{C}_{2} \mathrm{H}_{4},-0.372$ in $\mathrm{C}_{2} \mathrm{H}_{6}$, and -0.452 in $\mathrm{CH}_{4}$, while the charge on each H decreases as +0.188 in $\mathrm{C}_{2} \mathrm{H}_{2},+0.140$ in $\mathrm{C}_{2} \mathrm{H}_{4},+0.124$ in $\mathrm{C}_{2} \mathrm{H}_{6}$, and +0.113 in $\mathrm{CH}_{4}$. The $\mathrm{C}-\mathrm{H}$ overlap population is remarkably constant, ranging from 0.820 to 0.812 in $\mathrm{C}_{2} \mathrm{H}_{2}$ and $\mathrm{C}_{2} \mathrm{H}_{4}$, respectively, and from 0.785 to 0.790 in $\mathrm{C}_{2} \mathrm{H}_{6}$ and $\mathrm{CH}_{4}$. These small variations in overlap population are in accord with the bond lengths of 2.002 au in $\mathrm{C}_{2} \mathrm{H}_{2}, 2.022$ au in $\mathrm{C}_{2} \mathrm{H}_{4}, 2.066$ au in $\mathrm{CH}_{4}$, and 2.082 au in $\mathrm{C}_{2} \mathrm{H}_{6}$. The total carbon-carbon overlap populations are 1.875 for $\mathrm{C}_{2} \mathrm{H}_{2}, 1.211$ for $\mathrm{C}_{2} \mathrm{H}_{4}$, and 0.710 for $\mathrm{C}_{2} \mathrm{H}_{6}$. Thus the $\mathrm{C}-\mathrm{H}$ overlap population seems reasonably independent of bonding on the other side of the C atom. Even in HCN the $\mathrm{C}-\mathrm{H}$ overlap population is 0.800 , the charge of C is -0.141 , somewhat lower than that in $\mathrm{C}_{2} \mathrm{H}_{2}$, and the charge on H is +0.216 , somewhat higher than in $\mathrm{C}_{2} \mathrm{H}_{2}$. Hence, HCN fits into the series as HCN , $\mathrm{C}_{2} \mathrm{H}_{2}, \mathrm{C}_{2} \mathrm{H}_{4}, \mathrm{C}_{2} \mathrm{H}_{6}, \mathrm{CH}_{4}$.

Wave functions with an orbital exponent of 1.0 on H , already available for $\mathrm{C}_{2} \mathrm{H}_{6}$, have also been computed for $\mathrm{NH}_{3}, \mathrm{CH}_{4}$, and $\mathrm{C}_{2} \mathrm{H}_{4}$. The $\mathrm{C}-\mathrm{H}$ overlap populations change only slightly with this change in the H exponent; however, the $\mathrm{N}-\mathrm{H}$ overlap population changes from 0.677 for exponent 1.2 to 0.640 for exponent 1.0. Surprisingly, the C-C overlap population in $\mathrm{C}_{2} \mathrm{H}_{4}$ changes from 0.791 to 0.853 on changing the H exponent from 1.2 to 1.0 . In each case the total energy is lower for an exponent of 1.2 on H . In $\mathrm{NH}_{3}$ the difference between the energy with 1.0 and 1.2 exponents is 0.0407 au ( 0.0136 au per H atom). For $\mathrm{CH}_{4}$ this difference is $0.0535 \mathrm{au}(0.0134$ au per H atom $)$, and for $\mathrm{C}_{2} \mathrm{H}_{6}$ it is $0.0777 \mathrm{au}(0.0130$ au per H atom). On the other hand, the difference in ethylene is 0.0723 ; this last result is in the same direction but is 0.0181 au per H atom in magnitude. Another very interesting study is the effect of changing the exponent of H on the barrier to rotation in ethane and such a calculation is in progress by Dr. R. M. Pitzer.

Several numbers have been computed from the ethane wave functions which, however, do not yet give a simple interpretation of the internal rotation barrier. The total noncylindrical charge ${ }^{39}$ obtained by numerical integration over the electron density in staggered ${ }^{2}$ ethane is 4.129 electrons (e) of which 2.320 e are in H ls orbitals including $\mathrm{H}-\mathrm{H}$ overlap regions, and 1.809 e are in $\mathrm{C}-\mathrm{H}$ overlap. (None are on C by symmetry.) Values for eclipsed ${ }^{2}$ ethane are 4.154 e (total) and 2.333 e (on H ). For our model with an H exponent of 1.2 , values for staggered ethane are 4.570 e (total) and 2.460 e (on H ). Thus the protons of $\mathrm{CH}_{3}$ do not seem to be moving in a sea of cylindrical electron density. The total charge (also obtained by numerical integration) which is spherically symmetrical about each H is 0.834 e (both staggered ${ }^{2}$ and eclipsed ${ }^{2}$ ) and 0.789 e (staggered, H exponent 1.2). Thus the H nucleus is well shielded. Hence we doubt that discussions of nuclear contributions separately from electronic contributions will yield a simple interpretation of the barrier. We note further that an increase in the $\mathrm{C}-\mathrm{C}-\mathrm{H}$ angle of only 0.0022 radian (less than zero-point amplitudes)

[^3]Table XXV. Wave Function of Ammonia

| MO | Orbital energy | $\mathrm{H} 1$ | H2 | H3 | Coeffici 1 sN | $\text { ts } \overline{2 \mathrm{sN}}$ | $2 \mathrm{p}_{2} \mathrm{~N}$ | $2 \mathrm{p}_{x} \mathrm{~N}$ | $2 \mathrm{p}_{y} \mathrm{~N}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $1 a_{1}$ | -15.5230 | 0.0050 | 0.0050 | 0.0050 | -0.9957 | -0.0225 | -0.0041 | 0 | 0 |
| $2 a_{1}$ | -1.1014 | -0.1554 | -0.1554 | -0.1554 | 0.2096 | -0.7390 | -0.1375 | 0 | 0 |
| $1 \mathrm{e}_{x}$ | -0.5824 | 0.5034 | -0.2517 | -0.2517 | 0 | 0 | 0 | 0.5901 | 0 |
| $1 \mathrm{e}_{4}$ | -0. 5824 | 0 | 0.4359 | -0.4359 | 0 | 0 | 0 | 0 | 0.5901 |
| $3 a_{1}$ | -0.3661 | -0.1247 | -0.1247 | -0.1247 | $-0.0830$ | 0.4506 | -0.8921 | 0 | 0 |
| $4 a_{1}$ | 0.5844 | 0.7114 | 0.7114 | 0.7114 | 0.1683 | -1.2708 | -0.5626 | 0 | 0 |
| $2 \mathrm{e}_{x}$ | 0.6905 | 1.0071 | $-0.5036$ | $-0.5036$ | 0 | 0 | 0 | $-1.0562$ | 0 |
| $2 \mathrm{e}_{4}$ | 0.6905 | 0 | 0.8722 | $-0.8722$ | 0 | 0 | 0 | 0 | $-1.0562$ |
|  | Electronic energy -67.9386 <br> Nuclear repulsion 11.9334 <br> Total energy -56.0052 |  |  | Kinetic energy | 56.3959 |  |  |  |  |

Table XXVI. Hamiltonian Matrix of $\mathrm{NH}_{3}$

| H 1 | H 2 | H 3 | N 1 s | N 2 s | $\mathrm{~N} 2 \mathrm{p}_{z}$ | $\mathrm{~N} 2 \mathrm{p}_{x}$ | $\mathrm{~N} 2 \mathrm{p}_{y}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| -0.563 | -0.348 | -0.348 | -1.008 | -0.887 | -0.221 | -0.434 | 0 |
| -0.348 | -0.563 | -0.348 | -1.008 | -0.887 | -0.221 | 0.217 | -0.375 |
| -0.348 | -0.348 | -0.563 | -1.008 | -0.887 | -0.221 | 0.217 | 0.375 |
| -1.008 | -1.008 | -1.008 | -15.519 | -3.742 | -0.025 | 0 | 0 |
| -0.887 | -0.887 | -0.887 | -3.742 | -1.859 | -0.119 | 0 | 0 |
| $-\mathbf{0 . 2 2 1}$ | -0.221 | -0.221 | -0.025 | -0.119 | -0.356 | 0 | 0 |
| $-\mathbf{0 . 4 3 4}$ | 0.217 | 0.217 | 0 | 0 | 0 | -0.328 | 0 |
| $\mathbf{0}$ | -0.375 | 0.375 | 0 | 0 | 0 | 0 | -0.328 |

Table XXVII. Hamiltonian Matrix of HCN

| H | C 1 s | C 2 s | $\mathrm{C}_{2} \mathrm{p}_{2}$ | $\mathrm{C} 2 \mathrm{p}_{x}$ | $\mathrm{C} 2 \mathrm{p}_{y}$ | N 1 s | N 2 s | $\mathrm{~N} 2 \mathrm{p}_{2}$ | $\mathrm{~N} 2 \mathrm{p}_{x}$ | $\mathrm{~N} 2 \mathrm{p}_{u}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| -0.591 | -0.853 | -0.794 | 0.443 | 0 | 0 | -0.070 | -0.188 | 0.207 | 0 | 0 |
| -0.853 | -11.333 | -2.655 | -0.008 | 0 | 0 | -0.003 | -0.694 | 1.157 | 0 | 0 |
| -0.794 | -2.655 | -1.526 | -0.232 | 0 | 0 | -0.933 | -1.035 | 0.793 | 0 | 0 |
| 0.443 | -0.008 | -0.232 | -0.868 | 0 | 0 | -1.574 | -1.022 | 0.391 | 0 | 0 |
| $\mathbf{0}$ | $\mathbf{0}$ | 0 | 0 | -0.212 | 0 | 0 | 0 | 0 | -0.413 | 0 |
| 0 | 0 | 0 | 0 | 0 | -0.212 | 0 | 0 | 0 | 0 | -0.413 |
| -0.070 | -0.003 | -0.933 | -1.574 | 0 | 0 | -15.644 | -3.760 | 0.031 | 0 | 0 |
| -0.188 | -0.694 | -1.035 | -1.022 | 0 | 0 | -3.760 | -1.936 | 0.279 | 0 | 0 |
| $\mathbf{0 . 2 0 7}$ | $\mathbf{1 . 1 5 7}$ | 0.793 | 0.391 | 0 | 0 | 0.031 | 0.279 | -0.657 | 0 | 0 |
| $\mathbf{0}$ | $\mathbf{0}$ | 0 | 0 | -0.413 | 0 | 0 | 0 | 0 | -0.199 | 0 |
| 0 | 0 | 0 | 0 | 0 | -0.413 | 0 | 0 | 0 | 0 | -0.199 |

causes the nuclear repulsion of eclipsed ethane to be equal to that of undistorted staggered ethane. Surely some small change ${ }^{40}$ in geometry of the $\mathrm{CH}_{3}$ group occurs during internal rotation. We therefore feel that a study of this change and the development of criteria which do not separate nuclear repulsions from electronic contributions are promising directions for further study of the nature of the barrier.
(40) E. B. Wilson, Jr., Proc. Natl. Acad. Sci. U. S., 43, 816 (1957).

Acknowledgment. We wish to thank R. M. Pitzer for a number of helpful interchanges of correspondence, and to acknowledge his and D. Merrifield's communication of results prior to publication. For financial support we wish to thank the Office of Naval Research and the U. S. Army Research Office (Durham). The program which computes the molecular dipole moment for a polyatomic molecule directly from the detailed wave function was written for the IBM 7094 by M. D. Newton and F. P. Boer.


[^0]:    (11) M. P. Barnett, "Methods of Computational Physics," Vol. II, Academic Press Inc., New York, N. Y., 1963, p 95; also M. P. Barnett and C. A. Coulson, Phil. Trans. Roy. Soc. (London), 243, 221 (1951).
    (12) Several of these programs have been submitted to the Quantum Chemistry Program Exchange at the University of Indiana.
    (13) I. Shavitt, "Methods in Computational Physics," Vol. II, Academic Press Inc., New York, N. Y., 1963, p 1.
    (14) I. Shavitt and M. Karplus, J. Chem. Phys., 36, 550 (1962); M. Karplus and I. Shavitt, ibid., 38, 1256 (1963); I. Shavitt and M. Karplus, ibid., 43, 398 (1965); C. W. Kern and M. Karplus, ibid., 43, 415 (1965).

[^1]:    (22) G. Herzberg, "Infrared and Raman Spectra," D. Van Nostrand Co., Inc., New York, N. Y., 1945, p 439.
    (23) J. J. Sinai, J. Chem. Phy s., 39, 1575 (1963).
    (24) R. M. Pitzer, personal communication of unpublished results.
    (25) B. J. Woznick, J. Chem. Phys., 40, 2860 (1964).
    (26) G. Herzberg, ref 22, p 398.
    (27) A. D. McLean, B. J. Ransil, and R. S. Mulliken, J. Chem. Phys., 32, 1873 (1960).

[^2]:    (28) G. E. Hansen and D. M. Dennison, J. Chem. Phys., 20, 313 (1952). (29) W. N. Lipscomb, "Boron Hydrides," W. A. Benjamin Co., New York, N. Y., p 166.
    (30) W. L. Clinton and B. Rice, J. Chem. Phys., 29, 445 (1958).
    (31) L. S. Bartell and B. L. Carroll, ibid., 42, 1135 (1965).

[^3]:    (39) Thus this integration sums over all charge density which is greater than the cylindrical minimum values, and thus is not equivalent to the first term of an expansion in cylindrical harmonics.

